Can We Quantize Gravity?

Yes We Can!

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Quantum Field Theory

Why Can't Quantum Mechanics Explain Gravity? (Op-Ed)

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Relativity versus quantum mechanics: the battle for the universe

Summary

QFT and GR in a nutshell

Quantum Field Theory

- Particles are associated with a field, e.g. electron field $\psi(\vec{x}, t)$ for electron, electromagnetic field $A(\vec{x}, t)$for photon and so on
- We can calculate expectation values using Path Integral

$$\langle F \rangle = \frac{\int \mathcal{D}\varphi F[\varphi] e^{i \int d^4x \mathcal{L}[\varphi]}}{\int \mathcal{D}\varphi e^{i \int d^4x \mathcal{L}[\varphi]}}$$

- Lagrangian specifies a quantum field theory. Eg. QED Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

- How do we get the Lagrangian? Through experiments and guesswork!

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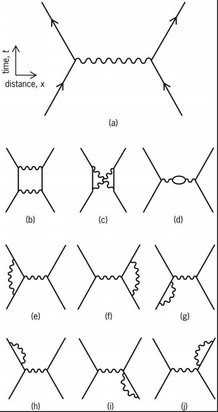
Feynman Diagrams

- For any scattering process, amplitude can be calculated by summing all relevant Feynman diagrams
- Feynman Diagrams can be computed by using rules derived from path integral
- Loop diagrams include integrating over loop momenta, and the result turns out to be infinite!



All one loop corrections to force between two electrons

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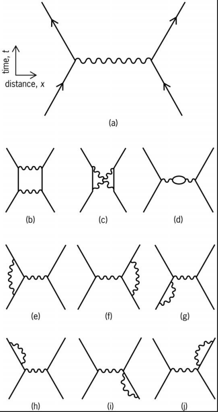

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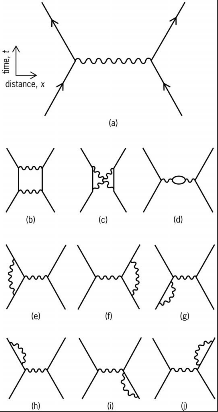

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General Relativity

- GR is given by Einstein-Hilbert action ($\kappa^2 = 32\pi G$)

$$S = \int \left[\Lambda + \frac{2}{\kappa^2} R[g] + \mathcal{L}_{\mathrm{M}} \right] \sqrt{-g} \mathrm{d}^4 x$$

- Λ is the Cosmological Constant, G is the same constant as in Newton's Law of Gravitation, $\mathcal{L}_M$ is the contribution from matter
- $R[g]$ is the 4 dimensional curvature of space-time. Spherical geometry has a positive curvature, hyperbolic geometry negative curvature and plane geometry zero curvature.
- The field associated with G.R. is the metric $g_{\mu\nu}$. Quantum Gravity starts with $g_{\mu\nu}$ as the relevant degree of freedom.
- Equation of motion: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \, T_{\mu\nu}$

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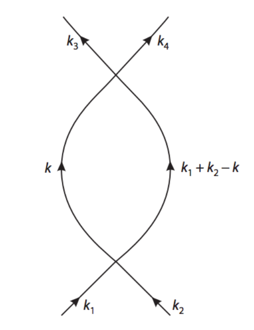
Renormalization and Problems with Gravity

Renormalization in ϕ^4 Theory

- $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4$
- This integral diverges! In fact if we put a cutoff Λ, the integral diverges as $\log(\Lambda)$.
- $\mathcal{M}(s,t,u;\Lambda) = -i\lambda + i\lambda^2\left[\log\left(\frac{\Lambda^2}{s}\right) + \log\left(\frac{\Lambda^2}{t}\right) + \log\left(\frac{\Lambda^2}{u}\right)\right]$ (We call the expression in square brackets $I(s,t,u;\Lambda)$)
- Scattering amplitude is a physical quantity, we can measure the above quantity in Lab for a given value of s,t,u.



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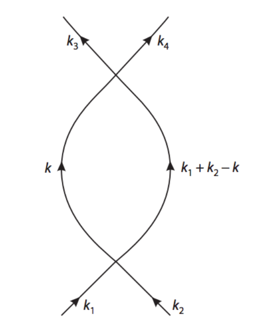

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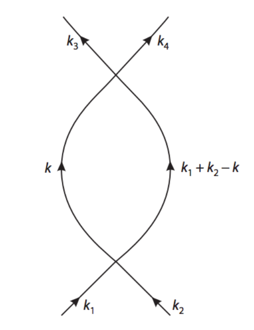

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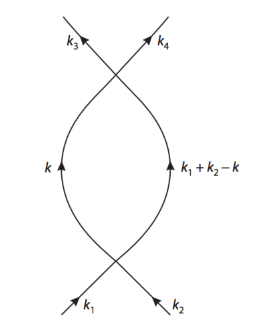

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- We know from experiments a value of $\mathcal{M}(s_0, t_0, u_0)$. More precisely $\mathcal{M}(s_0, t_0, u_0) \equiv -i\lambda_R = -i\lambda + i\lambda^2 \; I(s_0, t_0, u_0; \Lambda)$
- Renormalization: We say $\lambda(\Lambda)$ in such a way that the above equation is true for all Λ.
- This allows us to write $-i\lambda(\Lambda) = -i\lambda_R - i\lambda_R^2 \; I(s_0, t_0, u_0; \Lambda)$
- To calculate the amplitude at any other s, t, u, substitute $\lambda(\Lambda)$ in the expression for $\mathcal{M}(s, t, u)$ giving us

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- Note that this final expression has some quantities we have to determine from experiment, just like for classical electrodynamics.

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Renormalizable and Non Renormalizable Theories

- We saw that the dependence on cutoff disappeared in the final result. Was this a coincidence? In particular will the result hold to higher order loop corrections?
- If we can adjust a finite number of parameters such that all observable results are cutoff independent to all orders of loop corrections, then the theory is said to be renormalizable, otherwise it is said to be non-renormalizable.
- For renormalizable theories, we can made physical predictions to all orders of magnitude just by knowing the results of a finite number of experiments.
- The conventional view was that non-renormalizable theories would require infinite number of parameter adjustments, hence infinite number of observations.

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Criteria for Renormalizability

- We are using units in which $c = \hbar = 1$. In these units length and time have the same units and mass has units of inverse length. Let us count every dimension in units of mass.
- S is dimensionless (because of e^{iS}). Since $S = \int d^4x\mathcal{L}$, $[\mathcal{L}] = 4$. Thus all the terms in Lagrangian must have mass dimension of 4.
- Consider kinetic term like $\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$. This tells us that $[\phi] = 1$. Similarly we can deduce that $[\psi] = 3/2$ and $[A] = 1$. Coupling for scalar theory $[\lambda] = 0$
- It turns out that if the coupling has zero or positive mass dimensions then the theory is renormalizable.
- For Gravitation $[G] = -2$, so the theory is non-renormalizable!

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Effective Field Theories

Ignorance is Strength!

- We do not need to know entire theory before making predictions. For example Engineers do not need to know General Relativity before designing bridges
- Why is that the case? It turns out that the world works in such a way that physics at different scales can be separated from each other
- We can think of scales in terms of a parameter like energy or distance. At low energies, Newtonian Potential is a good enough. However at very high energies, say near a black hole event horizon, General Relativity is a better theory.
- We can think of Newtonian gravity as an effective field theory of General Relativity valid only in certain regime.
- However we know that General Relativity is not complete. It is also an effective theory of some higher theory, that we don't know yet. In this sense all physical theories are effective field theories.

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Ignorance is Strength!

- We do not need to know entire theory before making predictions. For example Engineers do not need to know General Relativity before designing bridges
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- We can think of scales in terms of a parameter like energy or distance. At low energies, Newtonian Potential is a good enough. However at very high energies, say near a black hole event horizon, General Relativity is a better theory.
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Quantum Effective Theories

- Effective Field theory is a way of recognizing the above paradigm in the context of Quantum Field Theories
- Effective field theories allow us to calculate experimental quantities, with errors parametrized by some relevant parameter δ.
- For example if we are working below the electroweak scale, we can consider the mass of W and Z Bosons as an expansion parameter.
- Calculations are done to some order of δ, known as power counting parameter. By choosing n in δ^n as large as we want, we can make our errors smaller. However large n involves higher order diagrams, which are harder to compute.
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Renormalization in Effective Field Theories

- In effective field theory, having a cutoff is not a mathematical trick. It expresses our belief that the theory is valid only up to certain energy scale.
- Cutoff has a physical interpretation. It is the scale beyond which new physics is expected to appear.
- The new physics might be new degrees of freedom, that is particles not present in the present theories. For example Fermi Theory of weak interaction breaks down when mass of Z boson becomes relevant. So it is natural to think of Λ as the mass associated with Z boson.
- Standard model itself is an effective field theory. In that case we expect some scale at which Standard model breaks down.
- In general in effective field theory, we want to write the most general Lagrangian associated with the degrees of freedom, consistent with known symmetries and gauge invariance.

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Effective Field Theory of Gravity

Lagrangian for GR

- Recall that Einstein Hilbert action is given by

$$S = \int \left[\Lambda + \frac{2}{\kappa^2} R[g] + \mathcal{L}_{\mathrm{M}} \right] \sqrt{-g} \mathrm{d}^4 x$$

- We quantize GR by background field method in which

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa \eta_{\mu\nu}$$

- Expanding the Lagrangian in $\eta_{\mu\nu}$, we get

$$\frac{2}{\kappa^2} \sqrt{g} R = \sqrt{\bar{g}} \left\{ \frac{2}{\kappa^2} \bar{R} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \dots \right\},$$

- To get the Feynman Rules we need to do gauge fixing and add ghost fields.

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Feynman Rules for Gravity

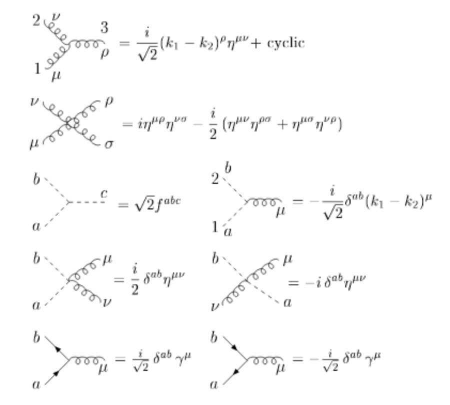

Figure: Feynman Rules

Renormalization?

- What happened to Renormalization? Isn't gravity non-renormalizable
- Infinities arise in a theory because we include energies all the way to infinity. However in the effective field theory point of view GR is valid only up to certain energy.
- The fact that GR is non-renormalizable does not matter as long as we are restricting ourselves to low energy.
- Corrections to GR can be included by higher order operators. Thus we modify our GR Lagrangian by

$$S = \int \left[\Lambda + \frac{2}{\kappa^2} R[g] + c_1 R^2[g] + c_2 R_{\mu\nu} R^{\mu\nu}[g] + \cdots + \mathcal{L}_{\mathrm{M}} \right] \sqrt{-g} \mathrm{d}^4 x$$

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Predictions from Effective Field Theory

$$\begin{aligned}
\mathscr{A}(++;++) &= \frac{i}{4}\frac{\kappa^2 s^3}{tu}\left(1+\frac{\kappa^2\, s\, t\, u}{4(4\pi)^{2-\varepsilon}}\frac{\Gamma^2(1-\varepsilon)\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\times\right.\\
&\times\left[\frac{2}{\varepsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\\
&\left.\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]\right)\\
\mathscr{A}(++;--) &= -i\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right)\\
\mathscr{A}(++;+-) &= -\frac{1}{3}\mathscr{A}(++;--) \qquad (29)
\end{aligned}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right)\\
&+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u}\\
&+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \qquad (30)
\end{aligned}$$

Predictions from Effective Field Theory

- In that equation + and - are graviton helicities. Only the first term is non-zero at tree level, other terms are suppressed by κ^2.
- The divergence is infrared in origin and can be cancelled by including gravitational bremsstrahlung.
- Except for infrared sector, there is no divergence and no unknown parameters are involved.
- No matter what the ultimate ultraviolet completion of the gravitational theory, the scattering process must have this form and only this form, with no free parameters, as long as the full theory limits to general relativity at low energy.

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Correction to Newton's Laws

$$V(r) = \frac{-GMm}{r}\left[1 + 3\frac{G(M+m)}{rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3} + \ldots\right]$$

Limitations to Effective Field Theory of GR

What is the catch?

- Effective field theories are expected to have limitations. At high energies new degrees of freedom might come that we have not accounted for. For example in QCD pions get replaced by quarks and gluons at high energy.
- String theory might be a completion of GR. However no matter what the UV completion is we expect the results to be the same at low energies.
- There might be other issues with GR. It is unknown whether GR is valid in the infrared limit. Indeed modification of GR at very low accelerations is a proposed solution to the dark matter problem.
- This effective theory has a problem dealing with non-local geometry of space-time. In particular it is not easy to see how we can deal with black holes.

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The End

Extra Slides

Renormalization in Classical Electrodynamics

- Classically we can calculate self energy of a sphere of radius r_e and charge q

$$E = m_{\text{em}} = \int \frac{1}{2} E^2 \, dV = \int_{r_e}^{\infty} \frac{1}{2} \left(\frac{q}{4\pi r^2} \right)^2 4\pi r^2 \, dr = \frac{q^2}{8\pi r_e}$$

  which becomes infinite as $r_e \to 0$

- The idea of Regularization: Let's not take the limit of $r_e \to 0$. Then we have $m_{\text{em}}(r_e)$, and a new parameter in theory r_e, called classical electron radius.
- However all efforts to observe structure of electron failed, electron does behave as a point charge.

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- Lorentz came with the idea of dividing the electron mass into mechanical mass and electromagnetic mass $m_{\text{tot}} = m_{\text{em}} + m_{\text{mech}}$
- If $m_{\text{mech}}(r_e)$, called counter-term, is allowed to be a function of r_e, we can vary in such a way that m_{mech} is finite. This process is called renormalization.
- We however lose our ability to calculate electron mass purely from electromagnetism, it is now an experimental value.
- Physical observables, like acceleration of an electron in presence of Electric field should not depend on r_e.
- We can compute the equation of motion of electron by assuming it is a sphere of r_e, taking into account the back-reaction, and one obtains the so called Abraham-Lorentz-Dirac equation.

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